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DEVELOP SEMI-EMPIRICAL MASS FORMULA BY ADDING A THERMAL TERM INDEPENDENTLY

Annotation: *In this paper, the best possible estimation of the semi-empirical mass formula (SEMF) coefficients was obtained, using the most accurate experimental masses values. A new individual thermal term $a_T f(A)$ has also been introduced in the SEMF. This term does not change the values of the previous SEMF coefficients, it*

reduces the standard deviation of the experimental data from the theory, that is, the experimental measurements have become more consistent with the SEMF, and it provides a method for calculating specific heat by knowing the Fermi's level energy of the nucleus.

Key Words: *semi-empirical mass formula, thermal term, least-squares method.*

ВЛИЯНИЕ АКВЕХОВОГО ЭКСТРАКТА СЕМЯН ХАРМАЛЫ НА НЕКОТОРЫЕ БИОХИМИЧЕСКИЕ И ГИСТОЛОГИЧЕСКИЕ ПАРАМЕТРЫ ПЕЧЕНИ У ХОМЯКОВ

Аннотация: *В настоящем документе была получена наилучшая возможная оценка коэффициентов полуидерной массы (SEMF) с использованием наиболее точных экспериментальных значений масс. В SEMF также был введен новый индивидуальный тепловой термин a_T f(A). Этот термин не изменяет значения предыдущих коэффициентов SEMF, он уменьшает стандартное отклонение экспериментальных данных от теории, то есть экспериментальные измерения стали более совместимыми с SEMF, и он обеспечивает метод расчета конкретного тепла, зная уровень энергии Ферми ядра.*

Ключевые слова: *полуразвердивая формула массы, тепловой термин, метод наименьших квадратов.*

1. Introduction:

In the area of nuclear physics model based on experimental data such as is common to many fields. These models have been confirmed by experimental data and must able to predict some other data. Semi-empirical mass formula (SEMF), known as Weizscker's formula or the Bethe–Weizscker formula in nuclear physic, is used for estimating the atomic mass as a function of mass number and atomic number. As the name implies, SEMF includes both empirical and theoretical parts; the theoretical part of this formula is obtained from the “liquid drop” model as proposed by George

Gamow containing some terms which were later developed by Niles Bohr and John Archibald Wheeler. The SEMF is formulated by a German physicist, Carl Friedrich von Weizscker, in 1935. The “liquid drop” model assumes the nucleus as a liquid drop together with its associated properties. According to the model, binding energy (BE) of the nucleus includes Volume Term, Surface Term, Coulomb Term, Asymmetry Term, and Parity Term. Theoretical calculations and data fitness are of the methods to determine coefficient of the terms in the “liquid drop” model [1]. The Weizscker's original nuclear binding energy formula [2], represented below, is

$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z), \quad (1)$$

Based on the liquid drop model, there is no thermal term, and this is a clear defect in it, because if the temperature is absent. then the nucleons of the nucleus would not behavior as a liquid drop molecules (the nucleons are static). Therefore, “liquid drop” model is not completely correct. This is what the researchers have attempted to rectify by introducing the effect of temperature in formula (1). For example [3-4], they suggested the following modification:

$$E_b(A,Z,T) = \alpha(T)A + \beta(T)A^{2/3} + \left(\gamma(T) - \frac{\eta(T)}{A^{1/3}}\right)\left(\frac{4t_\zeta^2 + 4\left|\left|t_\zeta\right|\right|}{A}\right)$$

$$+ \frac{0.8076Z^2R(0)}{A^{1/3}R(T)}\left(1 - \frac{0.7636}{Z^{2/3}} - \frac{2.29R(0)^2}{[R(T)A^{1/3}]^2}\right) + \delta(T)\frac{f(A,Z)}{A^{3/4}} \quad (2)$$

This modification did not decrease the standard deviation. The existence of relatively large differences between experimental and theoretical binding energies leads to a large standard deviation.

2. Method and results:

In a previous research [5], we obtained the results shown in Tables 1, 2, 3 and Figure 1, 2.

The Table 1 shows the SEMF coefficients. The Tables 2 and 3 show the differences between the experimental binding energies of the nucleon and the theoretical binding energies of the nucleon for odd and even nuclei $E_{BNexp} - E_{BNt}$□.

Table 1. Coefficients of SEMF.

Coefficient	Alonso [6]	Alonso [6]	Rohif [7]	Wapstr [8]	Mirzaei [1]	Our results for even nuclei	Our results for odd nuclei
a_V	15.8	15.76	15.75	14.1	15.519	15.8665	15.822
a_S	18.3	17.81	17.8	13	17.476	18.505968	17.973
a_C	0.714	0.711	0.711	0.595	0.664	0.70775	0.739
a_A	23.2	23.702	23.7	19	24.576	24.233985	20.886
δ-A	0	0	0	0	0	24.15235	0

Table 2. $E_{BNexp} - E_{BNt}$ for odd nuclei.

$E_{BNexp} - E_{BNt}$	A_i	$E_{BNexp} - E_{BNt}$	A_i	$E_{BNexp} - E_{BNt}$	A_i	$E_{BNexp} - E_{BNt}$	A_i
-0.00843768	161	0.024429076	95	-0.03614306	47	23.144	1
-0.01126027	165	0.025853499	99	-0.03339684	49	1.944	3
-0.00693411	167	0.002041573	103	-0.01104541	51	-0.01982477	7
-0.00407195	169	0.013892336	107	0.002521973	53	0.057460433	9
6.36E-05	171	-0.00571229	109	0.006369216	55	0.00965308	11
-0.00614433	175	0.003438794	111	0.013783966	57	0.186851432	13
-0.00302098	177	0.007318135	113	0.016490068	59	0.14391139	15
-0.00831995	181	0.02292562	115	0.020921432	61	-0.01506663	17
-0.00324938	183	-0.00182511	121	0.017690466	63	-0.15255374	19
0.000126223	185	-0.0062938	123	-0.01272188	65	-0.09265674	21
0.004064377	187	0.000534922	123	-0.02809984	67	-0.06033497	23
0.000567447	191	0.000837889	127	-0.02798337	69	-0.03581095	25
0.002413622	195	0.005091432	129	-0.04792125	71	0.000748103	27
0.011720336	197	0.006331524	133	-0.05325473	73	0.059375779	29
0.022033277	199	0.011835862	135	-0.04809366	75	0.044326715	31
0.022518542	205	0.016745708	139	-0.04296165	77	0.022318436	33
0.033373058	207	0.015839524	141	-0.03697269	79	0.014149604	35
0.031338537	209	0.024454376	143	-0.02505759	83	-0.04467388	37
-0.0352214	227	0.018275448	145	-0.01072174	85	0.008227014	39
-0.02742821	231	0.018438159	147	0.011153671	87	-0.08810522	41
-0.03188957	235	-0.00518859	151	0.035403321	89	-0.07920316	43
		-0.0109071	155	0.031656081	91	-0.07179393	45
		-0.01278083	159	0.020642812	93		

Table 3. $E_{BNexp} - E_{BNt}$ for even nuclei.

$E_{BNexp} - E_{BNt}$	A_i	$E_{BNexp} - E_{BNt}$	A_i	$E_{BNexp} - E_{BNt}$	A_i	$E_{BNexp} - E_{BNt}$	A_i
0.0186494	142	-0.04683	74	-0.040261	38	0.120555	4
0.011225	144	0.0423384	76	0.0176791	40	0.1398708	12
-0.013938	152	-0.024237	84	-0.069148	42	0.1106199	16
-0.017012	156	0.0127096	92	-0.000691	46	-0.166905	18
-0.013295	162	0.0103793	94	0.0762174	48	-0.062137	20
-0.009207	168	0.0198736	96	0.0061371	52	-0.135995	22
-0.007131	174	0.0040447	102	0.0047686	54	0.0051065	24
-0.00439	180	0.0051345	106	0.0212276	56	-0.064897	26
-0.004619	184	0.0012	112	0.0528855	58	0.0786942	28
-0.001281	190	-0.009961	120	0.0223657	60	-6.73E-05	30
0.0122558	196	-0.012067	124	-0.013981	64	0.0453502	32
0.0352766	204	0.0040007	132	-0.017746	66	-0.025225	36
-0.014511	234	0.0031025	136	-0.03762	70		

The Figure 1 and 2 shows a graph of the differences between the experimental binding energies of nucleon and theoretical binding energies in terms of the mass number for the odd and even nuclei. We note that the differences between the experimental binding energies of nucleon and the theoretical binding energies of nucleon are greater than the experimental error.

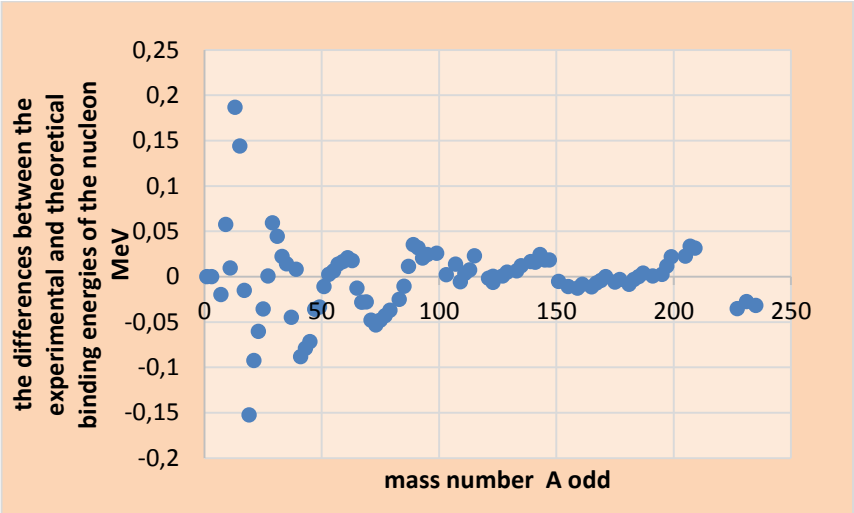

Figure 1. Differences between the experimental binding energies of nucleon and theoretical binding energies of nucleon for odd nuclei.

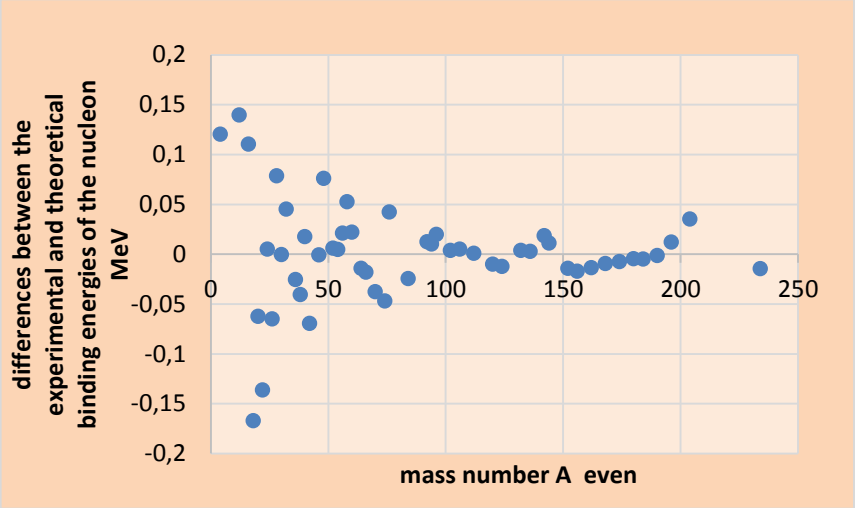

Figure 2. Differences between the experimental binding energies of nucleon and theoretical binding energies of nucleon for even nuclei.

The standard deviation of experimental values from the theoretical values for odd nuclei is σ = 0.0423 and for even nuclei σ = 0.053. The standard deviation is much greater than the experimental error in determining the binding energies of nucleon. Therefore, we attribute the large differences between the experimental binding energies of nucleon and the theoretical binding energies of nucleon to a certain physical amount, and this amount is somehow related to temperature. Therefore, Equation (1) can be written as follows:

$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} + a_T f(A) \pm \delta(A,Z) \quad (3)$$

where $a_T f(A)$ is a suggested term by us, and it is related to temperature, we call it the thermal term, so a_T is the thermal coefficient.

Based on Figures 1 and 2, we can suggest that the function $f(A)$ has the following form:

$$f(A) = u_0 e^{-u_1 A} \sin(u_2 A + u_3) \quad (4)$$

Where u_0 is amplitude parameter of damping harmonic oscillator, u_1 is damping parameter of damping harmonic oscillator, u_2 is frequency parameter of damping harmonic oscillator, and u_3 is the phase difference parameter of damping harmonic oscillator. These parameters are determined from fitting the function $f(A)$ with the differences in Tables 1 and 2.

By fitting the function $f(A)$ with the differences (using Mathcad program), we find the parameters shown in Table 4. Figures 3 and 4 show the extent of congruence between the proposed function (orange lines) and the $\delta E = E_{BNexp} - E_{BNt}$ differences (blue dots).

Table 4. Parameters values of $f(A)$ for odd and even nuclei.

Parameters	u_0	u_1	u_2	u_3
For odd nuclei	0.09	0.021	0.25	43.06
For even nuclei	-0.16	0.03	0.27	178.93

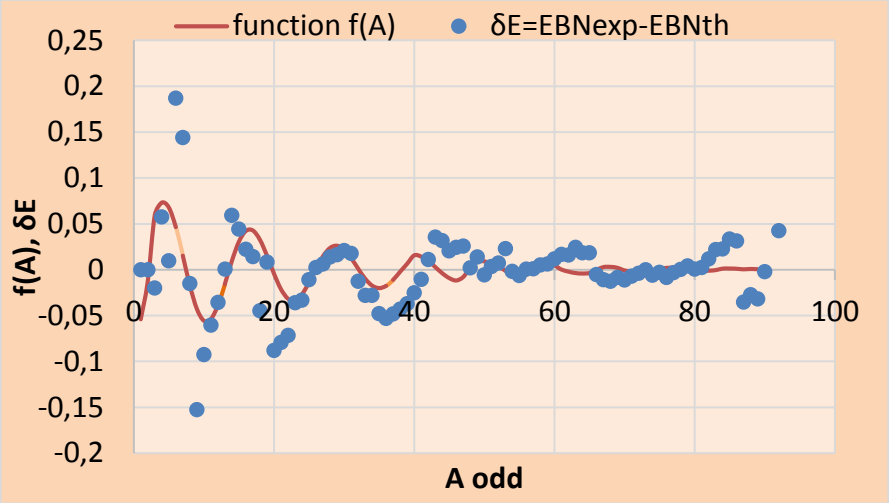

Figure 3. fitting for odd nuclei.

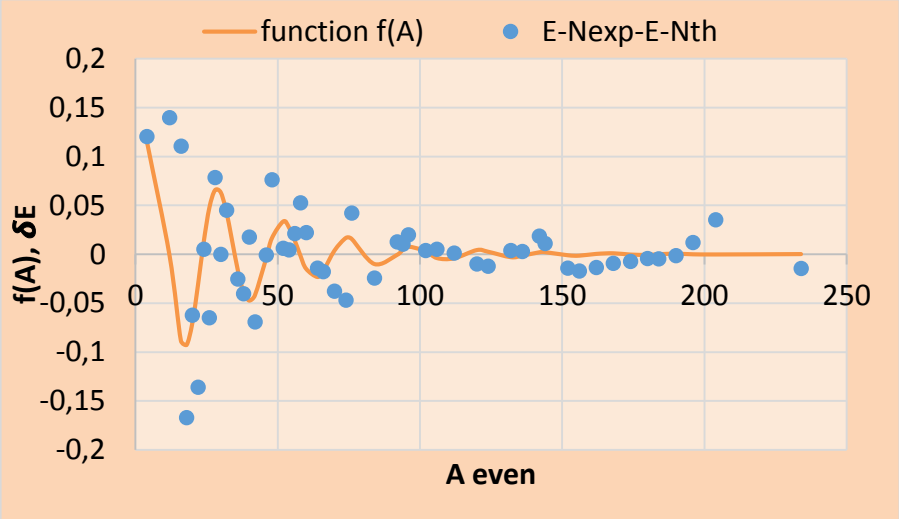

Figure 4. Fitting for even nuclei.

Thus, for odd nuclei

$$f_i(A_i) = 0.09.e^{-0.021A_i}.\sin(0.255A_i + 43.065) \qquad (5)$$

And for even nuclei

$$f_i(A_i) = -0.16\,.e^{-0.03\,A_i}.\sin(0.274A_i + 178.93) \qquad (6)$$

After adding the function $f\ (A)$, notice in Table 5 how the standard deviations have decreased.

Table 5. standard deviations before and after adding $f(A)$.

Standard deviation	for odd nuclei	for even nuclei
before adding $f(A)$	0.042	0.053
after adding $f(A)$	0.035	0.048

Application of the least squares method for determining the coefficients of SEMF after adding the thermal term for all single and even nuclei:

Experimental binding energy of the nucleus:

$$E_{Bexp}(Z,A) = \left[Z \cdot m_p + Nm_n - M\left({}^{A}_{Z}\mathrm{X}\right)\right]c^2 \text{ MeV} \qquad (7)$$

where Z is the number of protons, N is the number of neutrons, $A = N + Z$ is the mass number, $c^2 = 931.494061$ MeV/u is the square of the speed of light in unit of MeV / u, and $M\left({}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}\right)$ is the nuclide mass ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}$. The masses of the nuclides $\left({}^{\mathrm{A_i}}_{\mathrm{Z_i}}\mathrm{X}\right)$.

Theoretical binding energy of the nuclei: For odd nuclei,

$$E_{Bth}(Z,A) = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} + a_T f(A) \quad (8)$$

where a_V, a_S, a_C, a_A, a_T (which are unknown amounts) are the coefficients of the SEMF after we add the thermal term.

For even nuclei, the Equation (8) also contains the coupling term $\delta(A,Z)$.

Least squares method: congruence is best possible when the sum of squared deviations between empirical values and theoretical (error) S is minimized. The error is given by:

$$S = \sum_i (y_i - E_{Bth}(Z_i, A_i))^2 \qquad (10)$$

where

$$y_i = E_{Bexp}(Z_i, A_i) \qquad (11)$$

The matrix of coefficients [a] corresponding to a minimum error is obtained when the partial derivatives of the error with respect to the coefficients are zeros; Meaning $\partial S/\partial a = 0$:

$$\left.\begin{aligned}
\frac{\partial S}{\partial a_V} &= -2\sum_i A_i\ \left(y_i - E_{Bth}(Z_i, A_i)\right) = 0 \\
\frac{\partial S}{\partial a_S} &= \ \ 2\sum_i {A_i}^{2/3}\left(y_i - E_{Bth}(Z_i, A_i)\right) = 0 \\
\frac{\partial S}{\partial a_C} &= \ \ 2\sum_i \frac{Z_i(Z_i-1)}{{A_i}^{1/3}}\left(y_i - E_{Bth}(Z_i, A_i)\right) = 0 \\
\frac{\partial S}{\partial a_A} &= \ \ 2\sum_i \frac{(A_i - 2Z_i)^2}{A_i}\ \left(y_i - E_{Bth}(Z_i, A_i)\right) = 0 \\
\frac{\partial S}{\partial a_T} &= -2\sum_i A_i f(A_i)\ \left(y_i - E_{Bth}(Z_i, A_i)\right) = 0
\end{aligned}\right\} \Rightarrow (12)$$

After substituting Equation (8) in Equations (12), we get a set of

Exactly, the matrix equation (13) has the form:

$$[a] = \begin{bmatrix} a_V \\ a_S \\ a_C \\ a_A \\ a_T \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix}^{-1} \qquad (13)$$

Solving the matrix Equation (13) means finding the values of the elements of the matrix $[a]$. Where,

$$b_{11} = \sum_{\mathrm{i}} \mathrm{A_i}^2\ , b_{12} = \mathrm{b_{21}} = -\sum_i A_i^{\frac{5}{3}}\ , b_{31} = b_{31} = -\sum_{\mathrm{i}} \mathrm{Z_i(Z_i - 1)A_i^{\frac{2}{3}}}\ ,$$

$$b_{41} = b_{14} = -\sum_{\mathrm{i}} (\mathrm{A_i - 2Z_i})^2\ , \qquad b_{51} = b_{15} = 0.09\sum_i {A_i}^2 e^{-0.021A_i} \sin(0.25A_i + 43.06)$$

$$b_{22} = \sum_i A_i^{\frac{4}{3}}, b_{32} = b_{23} = \sum_i Z_i(Z_i - 1)A_i^{\frac{1}{3}}, \qquad b_{42} = b_{24} = \sum_i \frac{(A_i - 2Z_i)^2}{A_i^{\frac{1}{3}}},$$

$$b_{52} = b_{25} = -0.09\sum_i A_i^{5/3} e^{-0.021A_i} \sin(0.25A_i + 43.06)$$

$$b_{33} = \sum_i \frac{Z_i^2(Z_i - 1)^2}{A_i^{\frac{2}{3}}}\ , b_{43} = b_{34} = \sum_i \frac{Z_i(Z_i - 1)(A_i - 2Z_i)^2}{A_i^{\frac{4}{3}}}\ ,$$

$$b_{53} = b_{35} = -0.09\sum_i Z_i(Z_i - 1)A_i^{\frac{2}{3}}\ e^{-0.021A_i} \sin(0.25A_i + 43.06)\ ,$$

$$b_{44} = \sum_i \frac{(A_i - 2Z_i)^4}{A_i^2}, b_{54} = b_{45} = -0.09\sum_i (A_i - 2Z_i)^2\, e^{-0.021A_i} \sin(0.25A_i + 43.06),$$

$$b_{55} = 0.0081\sum_i (A_i e^{-0.021A_i} \sin(0.255A_i + 43.06))^2$$

$$c_1 = \sum A_i\, y_i \quad c_2 = -\sum_i A_i^{\frac{2}{3}}.\, y_i, c_3 = -\sum_i \frac{Z_i(Z_i-1)}{A_i^{\frac{1}{3}}} y_i,$$

$$c_4 = -\sum_i \frac{(A_i - 2Z_i)^2}{A_i} y_i, c_5 = 0.09 \sum_i e^{-0.021A_i} A_i \sin(0.25A_i + 43.06)\, y_i$$

Then, by numerically calculating the elements of the two arrays [b] and [c], and substituting them in Equation (13) we obtain the values of the coefficients for the odd and even ($\delta(0) = +a_P A^{-1/2}$) nuclei:

$$[a]_{\text{odd}} = \begin{bmatrix} a_V \\ a_S \\ a_C \\ a_A \\ a_T \end{bmatrix} = \begin{bmatrix} 15.829 \\ 17.992 \\ 0.739 \\ 20.89 \\ 0.631 \end{bmatrix} \qquad [a]_{\text{even}} = \begin{bmatrix} a_V \\ a_S \\ a_C \\ a_A \\ a_P \\ a_T \end{bmatrix} = \begin{bmatrix} 15.832136 \\ 18.399317 \\ 0.705177 \\ 24.17206 \\ 23.489915 \\ 0.440369 \end{bmatrix} \qquad (14)$$

3. Discussion:

Based on the above, we can visualize the nucleus model as a liquid drop as treating a nucleus located at a minimum temperature T_0. As for the room temperature, at which the experimental binding energy of the nuclei is determined, the nuclei have acquired the room temperature T_r. That is, the nucleus has acquired a thermal energy of $Q = CM(T_r - T_0)$, where M is the mass of the nucleus, and C is the specific heat of the nucleus material. The specific heat of the nucleus can be calculated if T_0 is known; That is, the temperature at which the nucleus becomes solid state, Thus it corresponds to the fermi level energy of the nucleus.

At a fixed temperature (such as room temperature), it is assumed that the nucleon is making vibratory motion of a constant amplitude. But the larger the number of nucleons (the mass number) in the nucleus, the greater the friction between nucleons. Consequently, it is expected the motion amplitude of nucleons will decrease as the mass number increases. This is indicated by figures (1) and (2).

When the nucleus is made up of one nucleon, its amplitude of its vibrational motion is great because there is no obstacle. When the nucleus is made up of two nucleons, each will block the movement of the other. Thus, the greater the mass number, the greater the disability among them. Therefore, the vibrational motion of nucleons gets dampened as their number increases. This is consistent with the function we suggested:

$$f(A) = u_0\, e^{-u_1 A} \sin(u_2 A + u_3)$$

- **Conclusions:**

Entering the function $f(A)$ in the SEMF, does not change the values of the coefficients of SEMF, The standard deviation of empirical data reduces theory. That is, the experimental measurements become more closely with SEMF. And this is what no one has accomplished before. It can provide a method for calculating specific heat by knowing the fermi level of the nucleus.

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